Sound field control in a reverberant room using the Finite Difference Time Domain method

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Outline

- Introduction
- The Finite Difference Time Domain method
- The Optimization Algorithm
- Simulation Results
- Conclusions
Introduction: Sound field reproduction

Sound field reproduction techniques aim to reproduce a measured or synthesized acoustic wave field in a different acoustic environment.

State of the art:

- Ambisonics
- Wave field synthesis
Introduction: multi-zone sound field control

Multi-zone sound field control: play sound in a specific area of the room (bright zone) while keeping other locations silent (dark zone)

- multi-zone sound field control pressure matching and sound field reproduction can both be optimization problems

In this presentation:

- Room acoustic physical model is added in the optimization (FDTD)
- Sparsity is used to optimize control source positions
The Finite Difference Time Domain Method

Wave Equation

PDE \[ \Delta p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = s \text{ on } \Omega \times \tau \]

BCs \[ \frac{\partial p}{\partial t} = -c \xi \nabla p \cdot n \text{ on } \partial \Omega \times \tau \]

ICs \[ \frac{\partial p}{\partial t} = \hat{p}_0, p = p_0 \text{ on } \Omega, \]

Approximate:

- the continuous scalar functions \( p \) and \( s \) on a uniform grid: \( p(x, y, t) \approx p(lX, mX, nT) = p_{l,m}^n \)
- the second order derivatives using centered finite differences: \( \frac{\partial^2 p}{\partial x^2} \approx \frac{p_{l+1,m}^n - 2p_{l,m}^n + p_{l-1,m}^n}{x^2} \)

Obtain a set of linear equations

\[ \mathbf{B} \mathbf{p} = \mathbf{s} \]

where \( \mathbf{B} \) is a sparse \( N_x N_y (N_t + 2) \times N_x N_y (N_t + 2) \) matrix and \( \mathbf{p} \) and \( \mathbf{s} \) are \( N_x N_y (N_t + 2) \) vectors containing the \( p_{l,m}^n \) and \( s_{l,m}^n \).
Optimization Algorithm

Optimization Problem

\[
\min_{s_c} \quad f = \frac{1}{2} \| F_p p - \tilde{p} \|_2^2 \\
\text{s. t. } \quad Bp = F_s^T s_c,
\]

Where:

- \( \tilde{p} \) is wanted sound pressure signals
- \( F_p \) is a selection matrix which selects the samples of \( p \) that correspond to the control points
- \( s_c \) is a \( N_s \cdot K_s \) vector containing the signals of the sought control sources
- \( F_s^T \) is an expansion matrix

Can be solved analytically using least square (LS):

\[
\min_{s_c} \frac{1}{2} \left\| F_p B^{-1} F_s^T s_c - \tilde{p} \right\|_2^2
\]

\[
s_c^* = -(G^T G)^{-1} G^T \tilde{p}.
\]

But LS problem is usually ill-posed! The matrix \( G^T G \), can be non-invertible. In order to avoid this, a regularization term can be added to the cost function.
### Optimization Algorithm: Regularization

**Optimization Problem**

\[
\begin{align*}
\min_{s_c} & \quad f = \frac{1}{2} \| F_p p - \tilde{p} \|_2^2 + \phi_a(s_c) + \phi_T(s_c) + \phi_{dz}(p) \\
\text{s. t.} & \quad Bp = F_s^T s_c,
\end{align*}
\]

- Tikhonov regularization still used to limit control source signals being too large.
- Newton method used to solve problem \( d^i = -(\nabla^2 f(s_c^i))^{-1} \nabla f(s_c^i) \)

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**Tikhonov regularization:**

\[
\phi_T(s_c) = \frac{\lambda_T}{2} \| s_c \|_2^2
\]

- penalizes large values of control source signals
- but evenly distribute the energy!

**Group sparsity-inducing regularization**

\[
\phi_0(s_c) = \lambda_0 \sum_{k=1}^{K_s} \| F_k s_c \|_0
\]

- minimizes the number of active control source positions
- NP-hard!!

**Group sparsity-inducing regularization atan:**

\[
\phi_a(s_c) = \sum_{k=1}^{K_s} \frac{2\lambda_a}{\gamma \sqrt{3}} \left( \text{atan} \left( \frac{1 + \gamma \| F_k s_c \|_2}{\sqrt{3}} \right) - \frac{\pi}{6} \right)
\]

- approximates well the behavior of the \( l_0 \)-norm
- continuous and differentiable at 0
- can enhance sparsity better than the \( l_1 \)-norm relaxation

**A dark zone can be enforced in a similar fashion as the regularization**

\[
\phi_{dz}(p) = \frac{\lambda_{dz}}{2} \| F_{dz} p \|_2^2
\]

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**Figure 1:** \( l_0 \)-norm of a scalar variable and its relaxation with \( \gamma = 10 \) and \( \lambda_a = 0.12 \).
Simulation Set-Up

“True” sound field generated by FDTD method:

- performance evaluated in 2-D highly reverberant room $T60 = 2.8s$

![Diagram showing control source candidate positions, control points, and dark zone points.]

- Different wavelet tested to see performance for different frequency ranges
- Simulations are performed for 0.5 seconds ($N_t = 500$)
- 22 candidate loudspeaker positions
Simulation Results: Low frequency wavelet

- No dark zone
- 4 speakers used

(a) $\lambda_a = 0.18\gamma, \lambda_{dz} = 0, \varepsilon = -40.3 \text{ (dB)}, \sigma_i$
Simulation Results: Low frequency wavelet

- with dark zone
- 5 speakers used

(b) $\lambda_a = 0.17 \gamma$, $\lambda_{dz} = 0.05$, $\varepsilon = -42$ (dB), $\sigma_l$
Simulation Results: “High” frequency wavelet

- no dark zone
- 7 speakers used

(c) $\lambda_a = 0.027 \gamma, \lambda_{dz} = 0, \varepsilon = -54.7$ (dB), $\sigma_h$
Simulation Results: Low frequency wavelet

- Dark zone active
- 10 speakers used

(d) $\lambda_a = 0.024\gamma$, $\lambda_{dz} = 0.05$, $\varepsilon = -48.1$ (dB), $\sigma_h$

- The more sources the better the accuracy
- Some sources acts as “sinks” → anechoic sound in reverberant environment
Conclusions

- Novel method for sound field reproduction and multi-zone sound field control
  - Relies on a physical model (FDTD)
- A non-convex sparsity regularization term
  - Optimize position of sound sources
  - Minimizes number of sound sources
- Simulations results:
  - suggest number of active sound sources increases with bandwidth
  - Anechoic sound reproduced in reverberant environment

- Future research:
  - Reduction of computational load, directional sources, application real scenario
Thank you for listening!